Chapter 4 Review

Mean Value Theorem (MVT) Work Sheet

1) MVT: Consider the function $f(x) = x^2$ on the interval $[1,3]$.

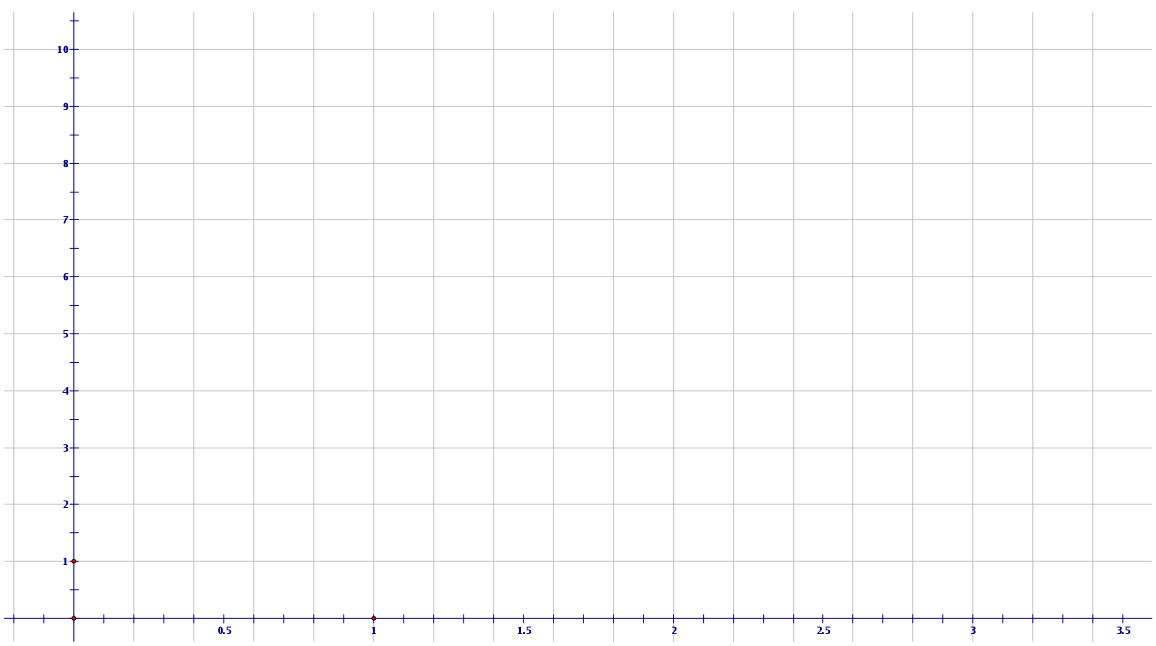
a) Graph the function neatly on the interval given.

b) Draw a line between the two endpoints of the interval and write the equation of the secant line.

c) Sketch a tangent line to the curve parallel to the secant line and approximate the x-coordinate of the point of tangency from the graph.

d) Use the MVT to calculate the x-coordinate of the point in part c.

$$\text{slope of secant } \frac{f(b) - f(a)}{b - a} = f'(c) \text{ slope of tangent}$$



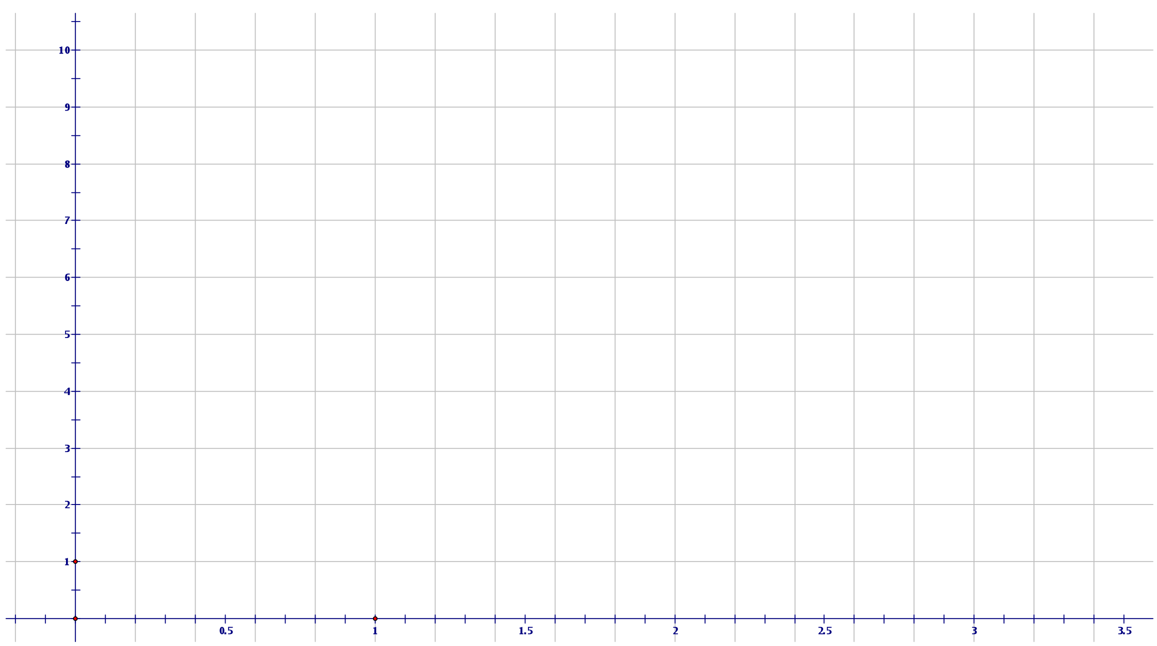
2) MVT for integrals: Consider the function $f(x) = x^2$ on the interval $[1, 3]$.

a) Graph the function neatly on the interval given.

b) Draw a horizontal line on the interval so that the area under the rectangle is approximately equal to the area under the function. Approximate the height of that rectangle (y-coordinate) and the location of that point (x-coordinate).

c) Use the MVT for integrals to find the average value (y-coordinate in part b) and the value of c (where the average value is located) (x-coordinate in part b).

Area under curve $\int_a^b f(x)dx = (b-a)f(c)$ Area of rectangle $\left(f(c)\ is\ f_{average}\right)$



3) Given velocity vs time data.

time	0	1	2	3	4	5	6	7	8
velocity	8.1	10.3	11.4	12.2	11.6	10.3	8.6	6.7	5.1

a) approximate $a(5.5)$

b) Is acceleration ever zero (justify)

c) Is velocity ever 10.7 (justify)

d) Find area under curve using 4 partitions

aa) Rectangles right

bb) Rectangles left

cc) Midpoint method

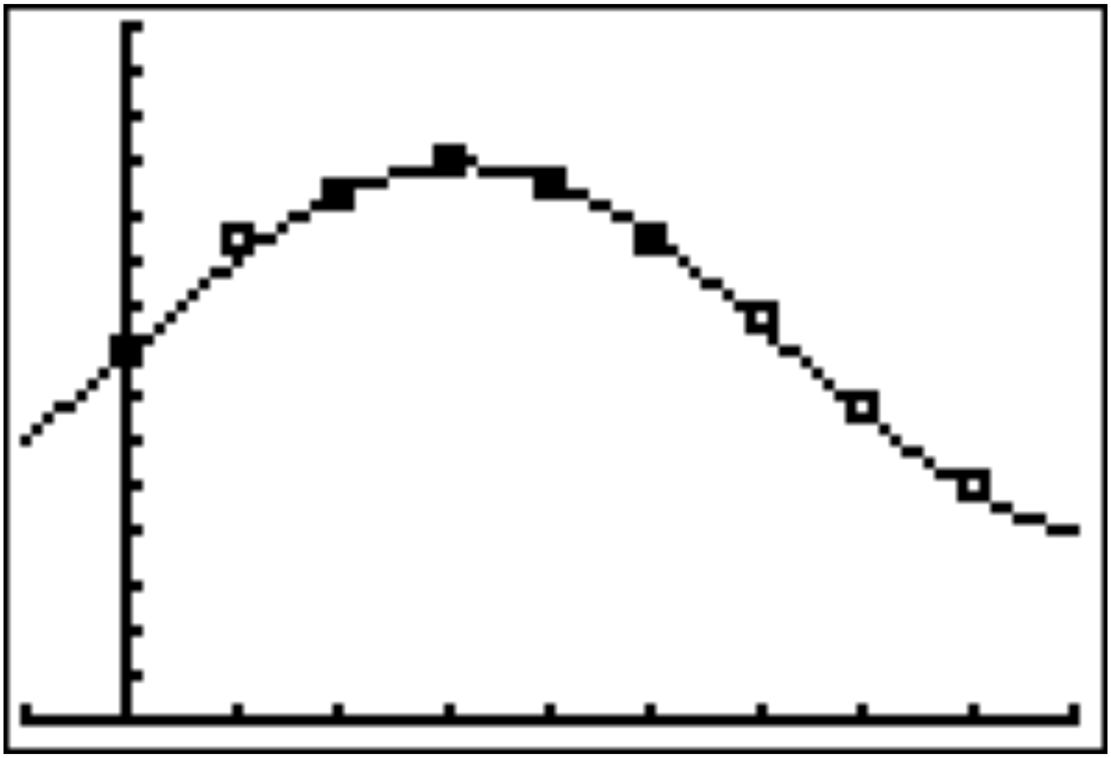
dd) Trapezoid method

e) What is the meaning of d

f) Find the average velocity (use any one from part d)

g) Find the average acceleration

4) A curve can be fit to the data in the first part. Using the curve $v(t) = 4\sin\left(\frac{t}{2}\right) + 8$, answer the questions by analysis and a calculator.



Calculator

aa) approximate $a(5.5)$

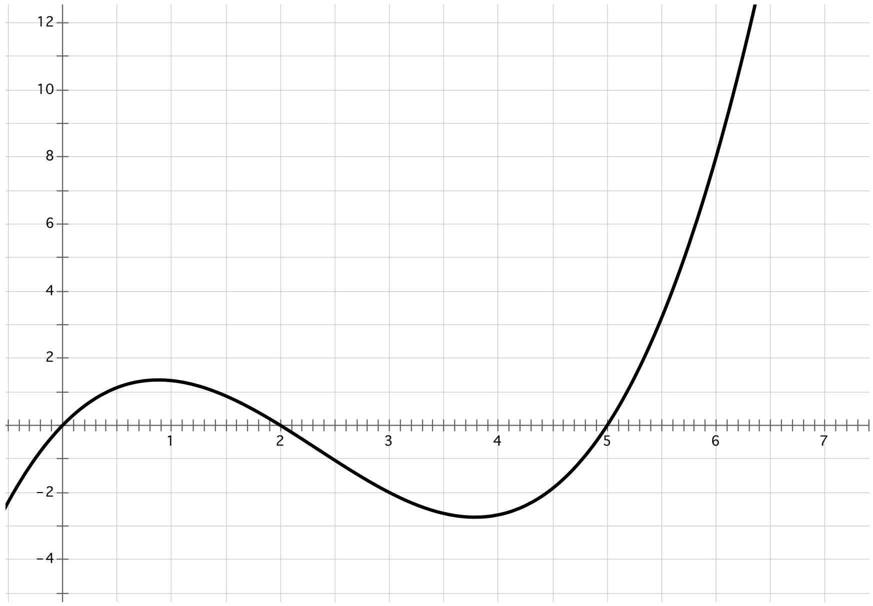
bb) Is acceleration ever zero (find acceleration and use calc)

cc) Is velocity ever 10.7

dd) Find area under curve using a definite integral

ee) Find the average velocity

5) Given the graph of $f(x)$ on the interval $[0,6]$.



$$g(x)=\int_0^x f(t)\,dt$$

a) Critical points on g, jlustify?

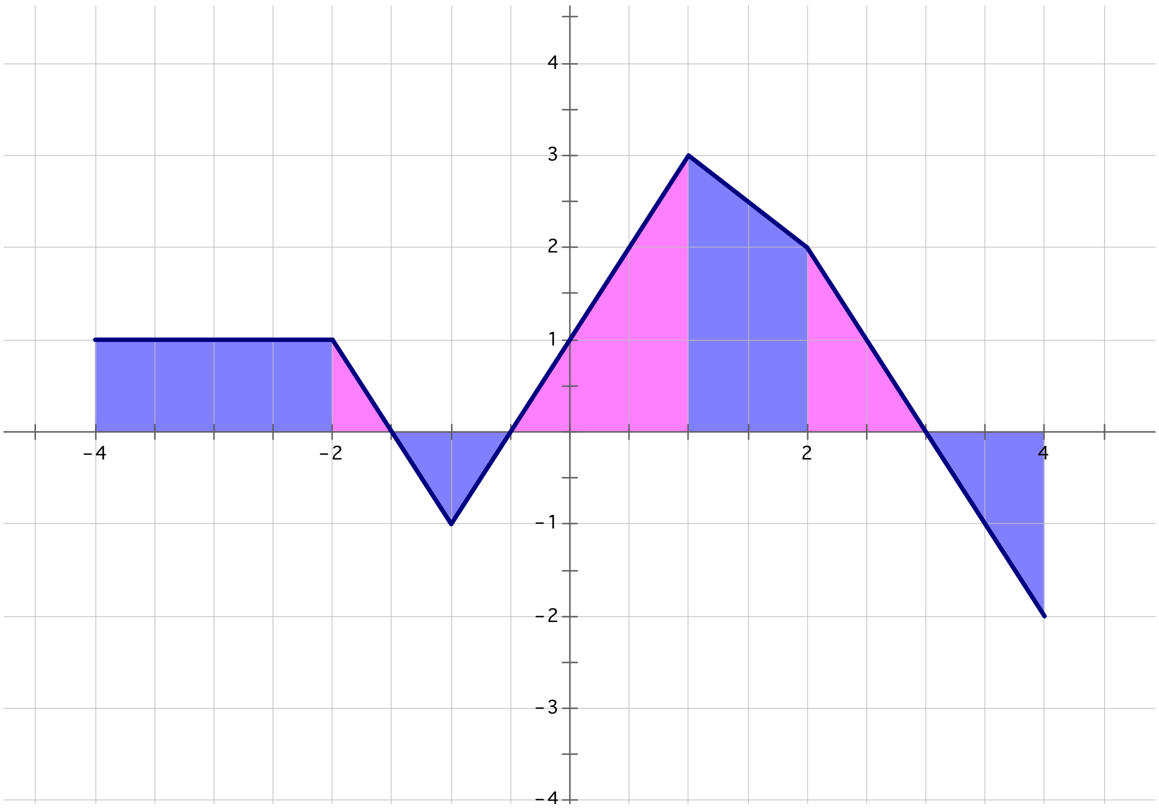
b) On what interval is g increasing / decreasing, justify?

c) Any Relative extreme, justify?

d) On what interval is g concave up / down, justify?

e) Any points of inflection, justify?

6) Answer the following questions from the graph of $f(x)$ below.



a) $\int_{-4}^{-2} f(x)dx =$

b) $\int_{-2}^{0} f(x)dx =$

c) $\int_{4}^{3} f(x)dx =$

d) $\int_{1}^{2} f(x)dx =$

e) $\int_{2}^{2} f(x)dx =$

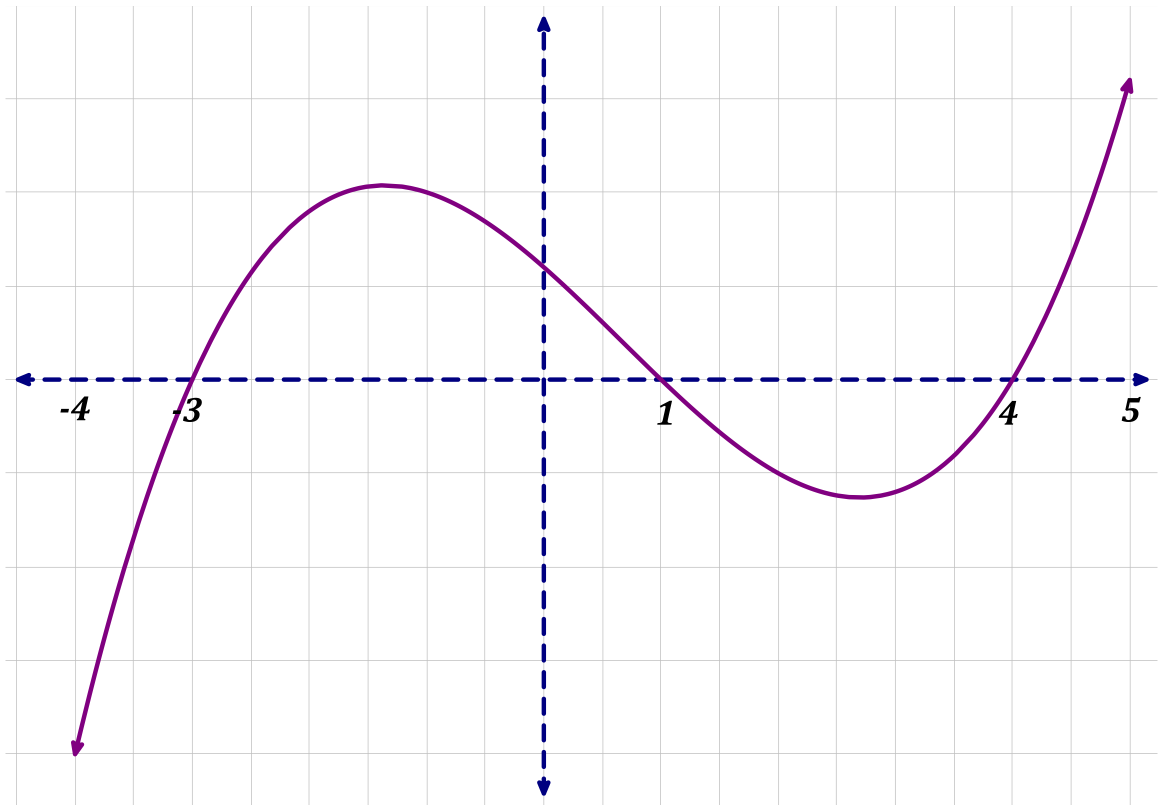
f) $\int_{-4}^{4} f(x)dx =$

g) $\int_{-1.5}^{-.5} f(x)dx =$

h) $\int_{-4}^{-1} f(x)dx =$

i) $\int_{3}^{2} f(x)dx =$

7) Given the information about $f(x)$ below, answer the following questions.



Given: $\int_{-4}^{-3} f(x)dx = -2$, $\int_{1}^{-4} f(x)dx = -4$, $\int_{4}^{1} f(x)dx = 3$, $\int_{4}^{5} f(x)dx = 2$

a) $\int_{-3}^{1} f(x)dx =$

b) $\int_{1}^{4} f(x)dx =$

c) $\int_{1}^{5} f(x)dx =$

d) $\int_{-3}^{4} f(x)dx =$

e) $\int_{4}^{-4} f(x)dx =$

f) $\int_{-4}^{5} f(x)dx =$

Derivative Worksheet

Basic derivatives: ($f'(x), \frac{dy}{dx}, y'$ **) Find** y'**.**

1) $y = 4$ 2) $y = 2x^3 + 3x^2 - 5x + 1$ 3) $y = \sin x$

4) $y = \cos x$ 5) $y = \tan x$ 6) $y = \csc x$ 7) $y = \sec x$

Use Algebra or Trig 1st, then find y':

1) $f(x) = x(3x^2 + 5x - 7)$ 2) $g(x) = \sin^2 x + \cos^2 x$ 3) $k(x) = \dfrac{5x^3 - 3x^2 + 2x}{x}$

4) $y = (\sin x)(\sec x)$ 5) $y = \dfrac{x^{\frac{5}{2}} - 3x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}}{\sqrt{x}}$ 6) $y = \dfrac{2x^2 - 3x + 1}{5}$

Basic derivatives with chain rule: $y' = operation(u)u'$

1) $y = (x^2 + 3x - 1)^3$ 2) $y = \sin 4x$ 3) $y = \cos x^2$

4) $y = \tan(3x + 1)$ 5) $y = \cot 2x$ 6) $y = \sec(3x - 4)$

Product rule using basic derivatives: $y = fg, \quad y' = fg' + gf'$

1) $y = \sin x \cdot x^2$

2) $y = \cos x \cdot 5x$

3) $y = \sin x \cdot \tan x$

4) $y = \cot x \cdot 3x^2$

5) $y = (2x+3)(x^2-1)$

6) $y = \csc x \cdot \sec x$

Quotient rule using basic derivatives: $y = \dfrac{f}{g}, \quad y' = \dfrac{gf' - fg'}{g^2}$

1) $y = \dfrac{\cos x}{x^3}$

2) $y = \dfrac{3x^2 - 3x + 1}{\tan x}$

3) $y = \dfrac{5}{\sin x}$

4) $y = \dfrac{\csc x}{x^4}$

5) $y = \dfrac{4x^3}{\cot x}$

6) $y = \dfrac{\sec x}{2x+1}$

Integral Worksheet

Basic Integrals (anti derivatives)

$$\int du = u + c \qquad \int k\, f(u)\, du = k \int f(u)\, du$$

$$\int \left(f(u) \pm g(u) \right) du = \int f(u)\, du \pm \int g(u)\, du$$

$\int u^n \, du = \dfrac{u^{n+1}}{n+1} + c$ Power rule good for any power except x^{-1} which is equal to $\dfrac{1}{x}$

examples: $\int x^3 \, dx = \dfrac{x^4}{4} + c,\quad \int x^{-3} \, dx = \dfrac{x^{-2}}{-2} + c,\quad \int x^{\frac{1}{2}} \, dx = \dfrac{2x^{\frac{3}{2}}}{3} + c$

1) $\int x^7 \, dx =$

2) $\int \sqrt[3]{x} \, dx =$

3) $\int \dfrac{dx}{x^{-3}} =$

U substitution and power rule & trig functions:

4) $\int x\left(2x^2 - 5\right)^3 dx =$

5) $\int \dfrac{3x^2}{5x^3 - 7} dx =$

$\int \sin u \, du = -\cos u + c$

12) $\int \sin x \, dx =$

13) $\int \sin 2x \, dx =$

14) $\int x \sin x^2 \, dx =$

$\int \cos u \, du = \sin u + c$

15) $\int \cos x \, dx =$

16) $\int -3\cos 4x \, dx =$

17) $\int x^3 \cos 2x^4 \, dx =$

$\int \sec^2 u\, du \quad same\, as \quad \int (\sec u)^2\, du = \tan u + c$

18) $\int \sec^2 5x\, dx =$

19) $\int x^4 \left(\sec x^5\right)^2 dx =$

$\int \csc^2 u\, du \quad same\ as \quad \int (\csc u)^2\, du = -\cot u + c$

20) $\int \csc^2(-3x)\, dx =$

21) $\int -2x \csc^2\left(x^2\right) dx$

$\int \sec u \tan u\, du = \sec u + c$

22) $\int \sec 5x\, tan 5x\, dx$

23) $\int x^5 \sec x^6 \tan x^6\, dx$

$\int \csc u \cot u\, du = -\csc u + c$

24) $\int \csc 3x \cot 3x\, dx$

Review Calculator Worksheet ______________

Approximate all answers to 3 decimals (whatever . 1 2 3)

Use the quadratic program to find the solutions to the following 3 questions.

1) $x^2 + 2.25 - 3x = 0$

Find the zeros of the following function by graphing the function and using the Calculate zero option.

2) $x^3 - 6x^2 + 3x + 9 = 0$

Solve the following by graphing the functions in Y1 and Y2, then use the Calculate intersect option.

3) $2x^3 - 3 = \sin x$

4) Find the absolute max/min of the function by graphing the function in its domain and then using the Calculate max/min option for the critical points and the table or trace for endpoints. Find $f'(-2.3)$ and

$$\int_{-4}^{1} f(x)\,dx \qquad f(x) = x^3 + 4x^2 + x - 5, \; [-4,1]$$

5) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{2x^2 - 4x + 5}}{3 - x}$

6) $\lim\limits_{x\to\infty} \sqrt{x^6 + 3x^3} - x^3$

7) $\lim\limits_{x\to 2} \dfrac{x^3 + x^2 - 10x + 8}{x^2 + 6x - 16}$